

Programme

Monday 3 April

10:30 Welcome with coffee - IRMA coffee room

Salle de conférences, IRMA

11:00 Bernard Leclerc I - University of Caen

14:00 Andrea D'Agnolo I - University of Padua

Tuesday 4 April

Salle de conférences, IRMA

11:00 Andrea D'Agnolo II

14:00 Bernard Leclerc II

Wednesday 5 April

Salle de conférences, IRMA

11:00 Pierre Baumann - IRMA, Strasbourg

14:00 Andrea D'Agnolo III

15:30 Bernard Leclerc III

Thursday 6 April

Petit Amphithéâtre, UFR de Mathématiques

10:30 Introduction - Catherine Florentz

Vice-President of Research,
University of Strasbourg

10:35 Gérard Laumon - University of Paris-Sud

14:00 Jean-Baptiste Teyssier - University of Leuven

15:00 Claude Sabbah - École Polytechnique, Paris

16:30 Introduction - Sylviane Muller
USIAS Chair in Therapeutic Immunology

16:35 Masaki Kashiwara - University of Kyoto

17:45 Reception - Salle Europe, MISHA

Series of talks in honour of Masaki Kashiwara

For more information:

[www.usias.fr/evenements/kyoto-lectures/
masaki-kashiwara](http://www.usias.fr/evenements/kyoto-lectures/masaki-kashiwara)

Organisation:

Nalini Anantharaman (IRMA, USIAS, Strasbourg)

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USIAS Kyoto lecture

Series of talks in honour of Masaki Kashiwara Kyoto University



3-6 April 2017

Institut de Recherche
Mathématique Avancée (IRMA)

7 rue René Descartes, Strasbourg

Kyoto lecture

Series of talks in honour of Masaki Kashiwara

Following Mikio Sato's pioneering work, Masaki Kashiwara developed, starting from his thesis in the seventies, the theory of D-modules. The point of view is to study a system of linear partial differential equations by using the methods of modern algebraic geometry and homological algebra. This change of paradigm led to a new field of research in mathematics, now called Algebraic Analysis. Among his striking results obtained in collaboration or alone, let us cite the involutivity of the characteristic variety of a D-module, the index theorem, the Riemann-Hilbert correspondence for holonomic D-modules in the regular case and recently, in the irregular case.

Kashiwara always kept a special interest for representation theory. One of his celebrated results is the proof, obtained in the early eighties in collaboration with Brylinski, of the Kazhdan-Lusztig conjecture (also proven independently by Beilinson and Bernstein). This conjecture proposed a mysterious equality between multiplicities in the representation theory of semi-simple Lie algebras and numerical data encoding singularities of Schubert varieties. In their proof the theory of D-modules provided a geometrical bridge connecting the two sides of the equality.

Motivated by the advances in the theory of integrable systems in the eighties, in particular the discovery of quantum groups, Kashiwara introduced crystal bases. These objects can be seen metaphorically as the residue of bases of representation of quantum groups when the temperature tends toward zero. There are now an important tool for the combinatorial description of representations of semi-simple Lie algebras, or more generally of Kac-Moody or even Borcherds algebras. They come together with bases called global crystal bases or canonical bases whose study has led to the current important theory of cluster algebras.



Bernard Leclerc is a Professor of Mathematics at the University of Caen, working in the domain of algebraic combinatorics and representation theory at the Laboratoire de Mathématiques Nicolas Oresme (LMNO). He is a Senior Fellow of the Institut de France (2010), and editor of the Journal of Combinatorial Theory A and the Mathematische Zeitschrift.

CANONICAL BASES AND CLUSTER ALGEBRAS

In 1990, G. Lusztig constructed a new basis of the positive part of the enveloping algebra of a simple Lie algebra, which he called the canonical basis. Its definition relied on the theory of quantum groups and the geometry of quiver varieties. In 1993, Berenstein and Zelevinsky formulated a conjecture on the dual of the canonical basis, that might lead to a more combinatorial description of this remarkable but rather mysterious basis.

In 2001, Fomin and Zelevinsky came up with a more precise conjecture in terms of a new class of rings called cluster algebras. The notion of a cluster algebra is elementary and combinatorial, and there are many examples, among which the dual of the positive part of the enveloping algebra of a simple Lie algebra. Fomin and Zelevinsky conjectured that the dual canonical basis contains all cluster monomials. This conjecture was proved in 2015 by Kang-Kashiwara-Kim-Oh, using categorification methods based on Khovanov-Lauda-Rouquier algebras.

The minicourse will try to give an accessible introduction to the Fomin-Zelevinsky conjecture, whose proof will be presented by M. Kashiwara.



Andrea D'Agnolo is Professor of Mathematics at the University of Padua. He works in the area of Algebraic and Microlocal Analysis.

ON THE RIEMANN-HILBERT CORRESPONDENCE

Hilbert's twenty-first problem (also known as the Riemann-Hilbert problem) asks for the existence of linear ordinary differential equations with prescribed regular singularities and monodromy. In higher dimensions, Deligne formulated it as a correspondence between regular meromorphic flat connec-

tions and local systems. In the early eighties, Kashiwara generalized it to a correspondence between regular holonomic D-modules and perverse sheaves on a complex manifold.

The analogous problem for possibly irregular holonomic D-modules (a.k.a. the Riemann-Hilbert-Birkhoff problem) has been standing for a long time. One of the difficulties was to find a substitute target to the category of perverse sheaves. In the 80's, Deligne and Malgrange proposed a correspondence between meromorphic connections and Stokes filtered local systems on a complex curve. Recently, Kashiwara and the speaker solved the problem for general holonomic D-modules in any dimension. The construction of the target category is based on the theory of ind-sheaves by Kashiwara-Schapira and uses Tamarkin's work on symplectic topology. Among the main ingredients of the proof is the description of the structure of flat meromorphic connections due to Mochizuki and Kedlaya.



Pierre Baumann is CNRS researcher at the Institut de Recherche Mathématique Avancée (IRMA) in Strasbourg. He works in the area of geometric representation theory of classical groups and algebraic combinatorics.

CRYSTALS AND BASES OF TENSOR PRODUCTS

In the early 90's, Kashiwara defined crystals as limits of bases of representations of quantum groups, and developed their theory in various situations. Subsequent works by Kashiwara, Lusztig, Littelmann, and Berenstein-Fomin-Zelevinsky elucidated the structure of these combinatorial objects. It was later discovered that Kashiwara's crystals also describe certain geometrical situations. For instance, as observed by Braverman-Gaietsgory, they occur in the geometric Satake correspondence. This theory, due to Lusztig, Ginzburg, Beilinson-Drinfeld, Mirković-Vilonen, and Ngô-Polo provides a construction of representations of a reductive group G from the geometry of the affine Grassmannian of the Langlands dual of G . We will study properties of bases that naturally arise in this context. This is joint work with S. Gaussent and P. Littelmann.



Gérard Laumon is a CNRS senior researcher at the University Paris-Sud. He studied at the École Normale Supérieure and Paris-Sud 11 University, Orsay. He was awarded the Silver Medal of the CNRS in 1987, and the E. Dechelle prize of the French Academy of the Sciences in 1992. In 2004 Laumon and Ngô Bảo Châu received the Clay Research Award for the proof of the Langlands and Shelstad's Fundamental Lemma for unitary groups, a component in the Langlands program in number theory. In 2012 he became a fellow of the American Mathematical Society.

EXOTIC FOURIER TRANSFORMATIONS OVER FINITE FIELDS

Independently, Braverman-Kazhdan (2003) and Lafforgue (2013) introduced a new approach to Langlands's functoriality involving Fourier transformations associated to Langlands transfer morphisms.

The Langlands functoriality has an analog over finite fields, which has been proved in full generality by Lusztig. So the Fourier transformation part of the above approach makes sense in that context.

In the talk, I will present some results that we have recently obtained with Emmanuel Letellier.

Jean-Baptiste Teyssier is a post-doctorate fellow of the Methusalem project in pure Mathematics at the Mathematics department of KU Leuven. Before this he held postdoctoral positions at the Freie Universität Berlin and the Hebrew University of Jerusalem.

SKELETONS AND MODULI OF STOKES TORSORS

In the local classification of differential equations of one complex variable, torsors under a certain sheaf of algebraic groups (the Stokes sheaf) play a central role. On the other hand, Deligne defined in positive characteristic a notion of skeletons for ℓ -adic local systems on a smooth variety, constructed an algebraic variety parametrizing skeletons and raised the question whether every skeleton comes from an actual ℓ -adic local system.

After some recollections on the Stokes phenomenon, we will explain how to use a variant of Deligne's skeleton conjecture in characteristic 0 to prove the existence of an algebraic variety parametrizing Stokes torsors. We will show how the geometry of this moduli can be used to prove new finiteness results on differential equations.



Claude Sabbah is CNRS senior researcher at the Centre de mathématiques Laurent Schwartz of the École Polytechnique in Palaiseau. He is working in the domain of linear differential equations in the complex domain and their applications to algebraic geometry. Claude Sabbah has been Vice-President of the French Mathematical Society (SMF). He was involved in the creation of the CEDRAM program, and the Journal de l'École Polytechnique, relaunched in 2013, of which he is now the journal manager.

IRREGULAR HODGE THEORY

Starting from the Riemann - resp. Birkhoff - existence theorem for linear differential equations of one complex variable, I will motivate on the example of hypergeometric - resp. confluent hypergeometric - equations the variant of Hodge theory called 'irregular Hodge theory', originally introduced by Deligne in 1984. I will also explain the interest of this theory in relation with mirror symmetry of Fano manifolds.



Masaki Kashiwara is Professor emeritus in Mathematics at the Research Institute for Mathematical Sciences, Kyoto University. He has made leading contributions towards algebraic analysis, microlocal analysis, D-module theory, Hodge theory, sheaf theory and representation theory. Together with Mikio Sato, Masaki Kashiwara established the foundations of the theory of systems of linear partial differential equations with analytic coefficients, introducing a cohomological approach that follows the spirit of Grothendieck theory of schemes.

CATEGORIFICATION OF CLUSTER ALGEBRAS VIA QUIVER HECKE ALGEBRAS

The notion of cluster algebras was introduced by Fomin-Zelevinsky. One motivation came from the multiplicative structure of upper global basis (or dual canonical basis). We use quiver Hecke algebras to categorify cluster algebras. Namely, the category of modules over quiver Hecke algebras has a structure of monoidal category. Its Grothendieck group has a cluster algebra structure. Simple modules correspond to the upper global basis, and cluster monomials correspond to simple modules.

$$\chi(X; F) = \#([\sigma_\phi] \cap CC(F))$$